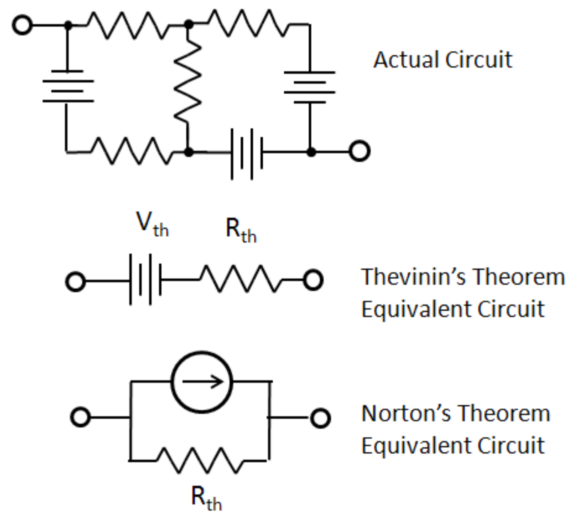
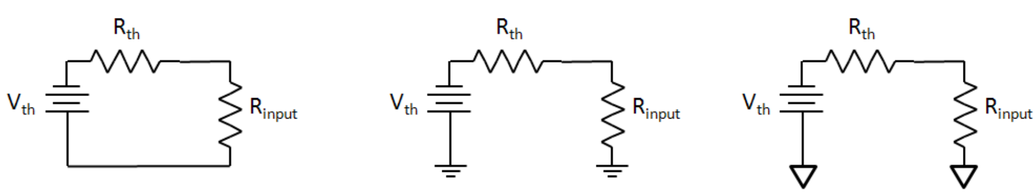
Input and Output Impedances of Resistive Circuits

Last time we saw that according to Thevinin's Theorem and Norton's Theorem, any kind of signal source (which we have arbitrarily modeled as a network of resistors and voltages sources) can be mapped to a either a perfect voltage source and a series resistor, or a perfect current source and a parallel resistor.



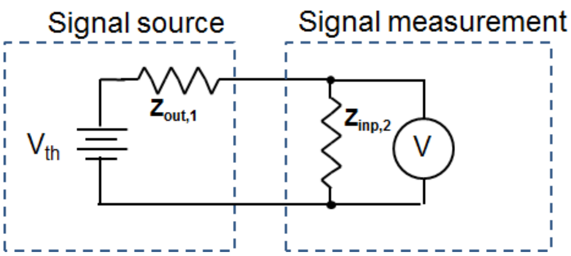
The fact that the top circuit can be modeled as either a voltage source (Thevinin's theorem) or a current source (Norton's Theorem) has an important implication: That any signal transducer can be thought of a either a voltage source or a current source. So, the question of "what is it" is one that is not always easy to answer, and in some cases a given tranducer can be used in either way, depending on how it is connected to the rest of the circuit. Whether a transducer is being used as a voltage source or a current source depends on the properties of what it is connected to – it is a property of the *system*, rather than property of an individual component.

When a signal source is connected to something else (such as an amplifier), the electrical properties can be modeled as a simple voltage divider. The figure below shows three equivalent ways or drawing the circuit, depending on whether we draw an explicit wire or show an implicit connection using the two common symbols for "ground" or "common".

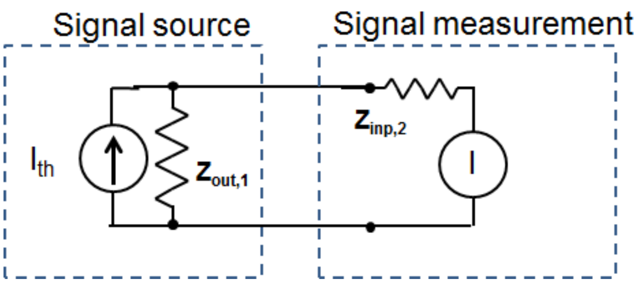


We we saw earlier, when we think of signal transducer in terms of its Thevinin equivalent circuit and we consider the input of a voltage measurement circuit (which could be an amplifier, filter, or other device that somehow measures or manipulates the signal), then the voltage divider equation tells us how accurately we are measuring the Thevinin voltage. We are now going to switch to the more general term of impedance (Z) instead of resistance R. The reason for this is to emphasize that the considerations below how true for both DC and AC signals, where the circuits can also include resistors, inductors, and capacitors. If we think of the signal transducer as as producing an output voltage V_{th} with an associated impedance $Z_{out, 1}$ that is connected to a signal measurement circuit with input impedance $Z_{in, 2}$, then the voltage measured as shown below will be: $V_{measured} = V_{th}\left(\frac{Z_{inp,2}}{Z_{out,1} + Z_{inp,2}}\right)$

Based on the voltage divider relationship, we will accurately measure the voltage produced by the signal source when $Z_{out,1} >> Z_{out,1}$



Similarly, if the think of the transducer as a current source and trying to measure its output current via a "real" ammeter (which is a perfect ammeter with an additional series resistance) then you can easily show that:



$$I_{measured} = I_{th}\left(\frac{Z_{out,1}}{Z_{out,1} + Z_{inp,2}}\right)$$

So, the most accurate current measurement is achieved when $Z_{inp,2} << Z_{out,1}$

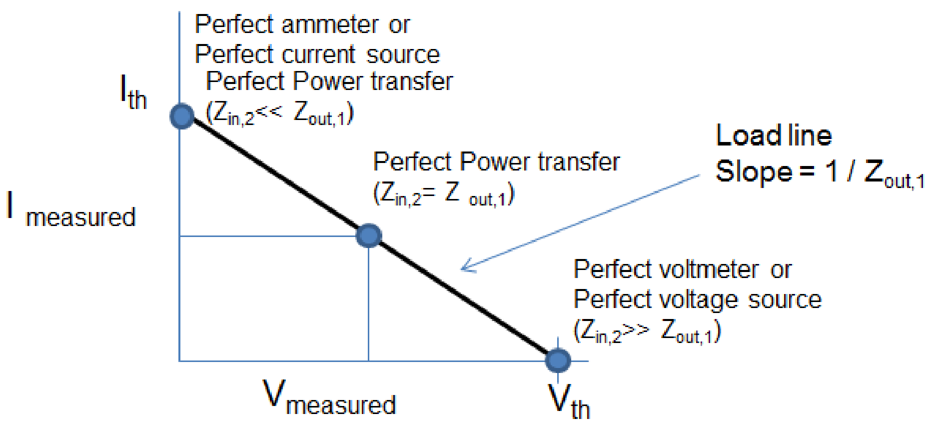
A perfect voltage source has $Z_{out} = 0$

A perfect current source has $Z_{out} = \infty$

A perfect ammeter (current measurement device) has $Z_{input} = 0$

A perfect voltmeter has $Z_{out} = 0$.

For any value of Zin, 2 we can represent the relationships via a load line.

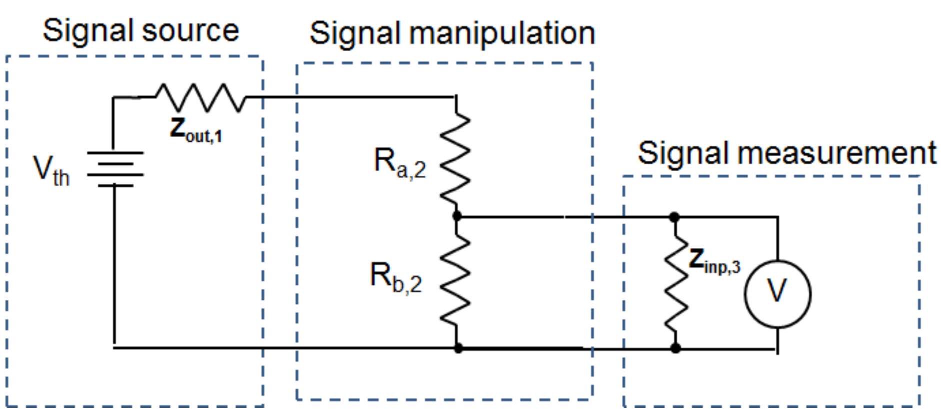


The load line says that if we have a perfect voltmeter so that the measurement instrument draws zero current, we measure V_{th}. If we short-circuit the output directly through a perfect ammeter (so that $V_{measured}=0$) we get I_{th}. For any value of $Z_{in,2}$ the measured voltage and/or measured current will fall along the load line, which has a slope just equal to $1/Z_{out,1}$.

In practice, the load-line concept provides one possible method for measuring the output impedance of an instrument: By measuring V_{th} and I_{th} separately, and noting that $R=V_{th}/I_{th}$

Since we are dealing with circuits that are linear (current is always proportional to voltage), another way of representing the relationship is via a "load line":

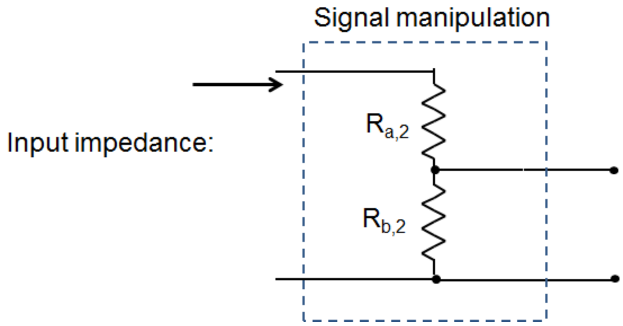
A more complicated example:



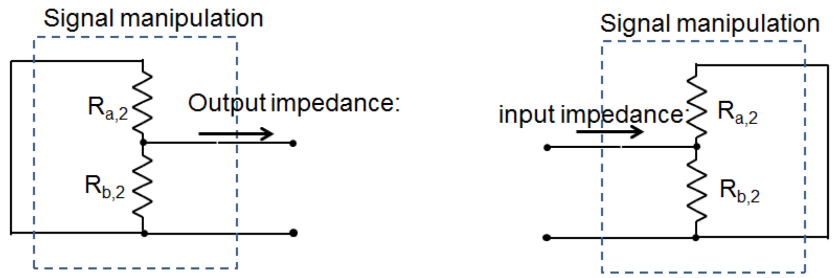
Here we have the more common situation, where we have a signal source, a module the somehow manipulates the signal (and therefore has both an input and an output), and then a signal measurement stage 2. This particular signal manipulation module is basically a simple attenuator: the resistors Ra,2 and Rb,2 lead to an output voltage that is smaller than the input voltage Vth according to the voltage divider equation. This circuit is actually quite commonly used.

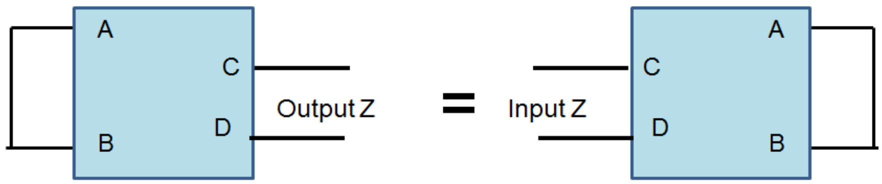
Ideally, in a situation like this we would want $V_{measured} = V_{th}\left(\frac{R_{a,2}}{R_{a,2}+R_{b,2}}\right)$ where the user would choose specific values for $R_{a,2}$ and $R_{b,2}$. In order to solve things absolutely correctly, we would have to solve Kirchoff's Laws for the complete system (we could, for example, use Kirchoff's Laws to combine the signal source and signal manipulation modules and come up with a new Thevinin equivalent circuit. A simpler approximation often suffices. We can, for example, assume that theintended circuit will have $Z_{inp,3}$ very large and $Z_{out,1}$ very small.

With this assumption, we can calculate the *input impedance* of second module (the signal manipulation module) as: $Z_{in,2} = R_{a,2} + R_{b,2}$



To calculate the output impedance is a little tricky. Remember that the output impedance describes the I vs. V response at the ouput when the circuit is connected to both the third and first modules. Most importantly, when calculating the output impedance of circuit 2 we assume that the input is connected to a signal transducer with a low resistance ($V_{th} << R_{a,2}+R_{b,2}$). So, to calculate the output impedance of the second module, we want to short the input leads and then calculate the I vs. V response at the output. How do we do that? It turns out that this is exactly equivalent to flipping the circuit around (swapping input for output), and the measuring the input resistance in the same way that we did before.





Measuring output impedance of passive circuits (R, L, C) → To calculate the output impedance you short the inputs and then treat the output just as if it was an input. This procedure works experimentally too, and you will use this in the next problem set. Note that procedure does not work with active circuits (that is, circuits with internal power supplies such as most real amplifiers) because they will usually become very unhappy if you start applying voltages to their inputs. If the circuits contain resistors only (no capacitors or inductors) then you can use a simple digital volt meter as shown below: